FROM LECTURE ON 5th OCTOBER

SVMs (Support Vector Machines).

- Linear Classifier
- MAXIMUM MARGIN HYPERPLANE

- Supports vectors = Points on Margin
- Algorithm for finding MMH
- Handle outliers.
- Map data onto high-dimension "feature-space"
  ↳ using '<u>Kernel Trick</u>'

ϕ : input data-point ⟶ point in feature-space

$\mathbb{R}^{20?} \longmapsto \mathbb{R}^{1xxxx}$ (maps to)

$\underbrace{\vec{x}^{(1)}, \ldots, \vec{x}^{(n)}}_{\text{data points}}$

find $\vec{w}$

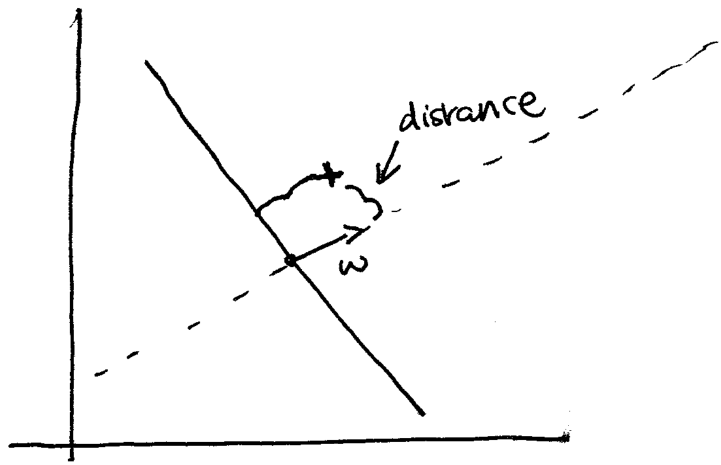
such that

$\text{sign}(w \cdot x) = c \iff w \cdot cx > 0$

desired outputs

$c^{(1)}, \ldots, c^{(n)}$

$c = \{-1, +1\}$



if $\|w\| = 1$
dist $\underline{x}$ to seperating plane is $w \cdot x$

$$\frac{w}{\|w\|} \cdot cx \geq \text{margin}$$

$$\downarrow$$

$$w \cdot cx \geq \text{margin} \cdot \|w\|$$

$$\downarrow$$

$$w \cdot cx \geq 1$$

$$\downarrow$$

Find **w** such that
(1)
(2) max $\underline{\underline{\|w\|}} \iff \|w\|^2 \iff \sum_i w_i^2$

To find MMH:

$\Rightarrow \max_{w} \sum_i w_i^2$ such that $\forall p \quad c^{(p)} w \cdot x^{(p)} \geq 1$

Support Vectors $= \{ x^{(p)} \mid c^{(p)} w \cdot x^{(p)} = 1 \}$

HANDLING OUTLIERS

For points on the wrong side of the margin:

$$c^{(p)} w \cdot x^{(p)} \geq 1 - \xi_p$$

$$\xi_p \geq 0$$

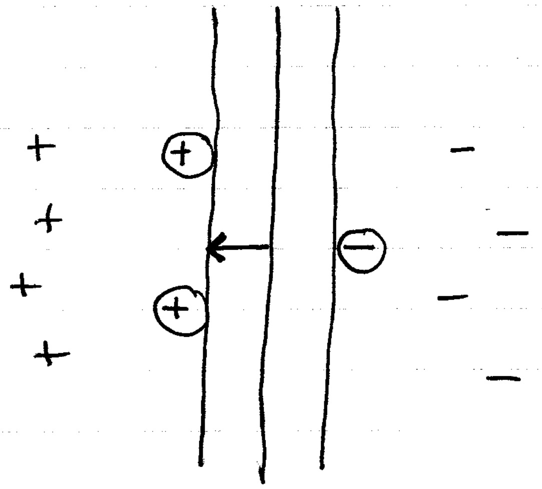
$\max_{w_i \alpha} \sum_i w_i^2 - \lambda \sum_p \xi_p$ such that

$\to \max_{w_i \xi} \quad w \cdot w - \lambda \sum_p \xi_p$

such that $\dfrac{c^{(p)}(w \cdot x^{(p)})}{c^{(p)}\left(\sum_p c^{(p)} \alpha_p x^{(p)}\right) \cdot x^{(p)}} \geq 1 - \xi_p, \quad \xi_p \geq 0$

$\downarrow$

$$\left(\sum_p c^{(p)} \alpha_p x^{(p)}\right) \cdot \left(\sum_p c^{(p)} \alpha_p \cdot x^{(p)}\right)$$



$$w = \sum_p c^{(p)} \alpha_p x^{(p)} = \sum_{p \mid c^{(p)}=+1} \alpha_p x^{(p)} - \sum_{p \mid c^{(p)}=-1} \alpha_p x^{(p)}$$